ACCURACY OF CALCULATING FLOWS UPON DISAPPEARANCE OF DISCONTINUITY BY THE METHOD OF LARGE PARTICLES

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The numerical solution of the problem of disappearance of discontinuity is compared in the univariate case with the accurate solution for the discontinuity of an arbitrary amplitude and with the calculation in Lagrange coordinates, and in the spatial case with the calculations on grids that differ in details.

At present, in solving a number of problems of aero- and gasdynamics [1-4], simulating non-steady-state explosive processes [5, 6], the dynamics of plasma erosion flares [7, 8], wave processes in multiphase media [9], various non-steady-state jet flows of gas and plasma [10-13], and other problems of engineering physics, the numerical method of large particles [1] is used. To a certain extent, the accuracy of the results obtained by this method was investigated in [1-4, 6, 9, 11-13] where the obtained numerical results were compared with calculations carried out by other methods, with experimental data, and with the accurate solutions of modeling problems However, in these and in many other works (as an example of univariate calculations we might point out [14, 15]), which investigated in detail the accuracy of various finite-difference approaches, the question of the influence of the numerical values of the parameters to be determined, of the intensity of the process, of the thermodynamic state of the medium on the accuracy of the calculation was not analyzed. The present article represents an attempt to study the accuracy of the realization of the method of large particles on the example of actual nonsteady-state unidimensional and two-dimensional flows of a monatomic ($\gamma = 5/3$) and polyatomic ($\gamma = 1.2$) ideal gas, due to the disappearance of discontinuity, in a broad range of changes of density and pressure of the environment.

We will examine the problem of disappearance of discontinuity in an infinite pipe with constant cross section situated in the direction of the z axis and filled with an ideal gas. When $t > 0$, a flow forms whose solution [6] is known and consists of a shock wave and a rarefaction wave which move in opposite directions (Fig. 1).

The results of the numerical solution obtained by the method of large particles for $p_3 = \rho_3 = 1$, $p_0 = \rho_0 = 10^{-1}, 10^{-3}, 10^{-6}$ with $\gamma = 1.2$ and 5/3 are presented in Figs. 2 and 3 at successive instants (all the magnitudes in the text are dimensionless). It can be seen that the accuracy of the calculation greatly changes in dependence on the initial discontinuity of density and pressure, and also on the thermodynamic properties of the gas, i.e., the magnitude of γ. The relative error of the calculated and of the accurate values of the magnitudes at the shock front (these are presented in Table 1) changes in dependence on the position of its theoretical front which does not coincide with the analytical front (Figs. 2 and 3). It can thus be seen that for a gas with $\gamma = 5/3$, the numerical solution is closer to the accurate one when the density and pressure gradients are small, and for a gas with $\gamma = 1.2$ when they are large. Thus influence of the magnitude of γ can be explained as follows.

TABLE 1. Analytical Values of the Gasdynamic Magnitudes at the Shock Front for Gases with $\gamma = 1.2$ and 5/3

p_0	u	p	ρ	D
		$\gamma=1,2$		
10^{-6}	6,20	$4,43\cdot10^{-5}$	$8,83\cdot10^{-6}$	6,99
10^{-3}	3,28	$1,38\cdot10^{-2}$	$6,17\cdot10^{-3}$	3,91
10^{-1}	1,12	$3,08\cdot10^{-1}$	$2,48\cdot10^{-1}$	1,87
		$\gamma=5/3$		
10^{-6}	3,44	$1,79\cdot10^{-5}$	$3,32\cdot10^{-6}$	4,93
10^{-3}	2,35	$9,45\cdot10^{-3}$	$2,88\cdot10^{-3}$	3,60
10^{-1}	0,88	$2,76\cdot10^{-1}$	$1,78\cdot10^{-1}$	2,00

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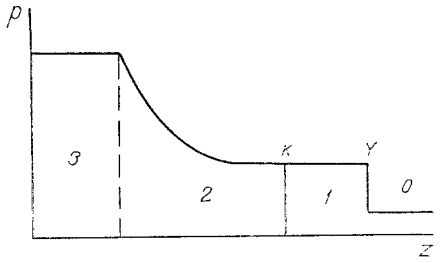

Fig. 1. Qualitative pattern of the disappearance of discontinuity: 0) region of cold gas; 1) region of gas compressed by the shock wave Y; 2) region of gas in the rarefaction wave; 3) hot gas; K) contact boundary.

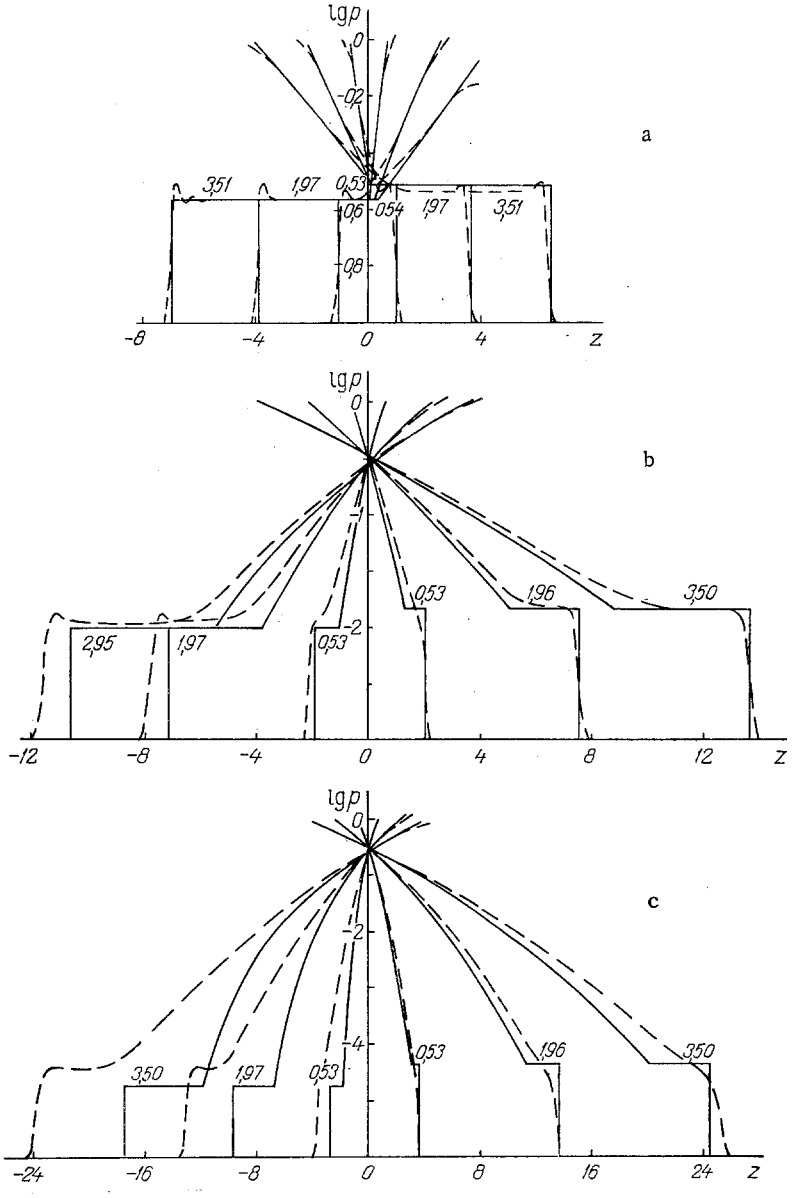

Fig. 2. Accurate (solid lines) and calculated (dashed lines) pressure profiles at successive instants for gas with $\gamma = 5/3$ (to the left of the axis of ordinates) and for gas with $\gamma = 1.2$ (right): a) $p/p_0 = 10$; b) 10^3; c) 10^6.

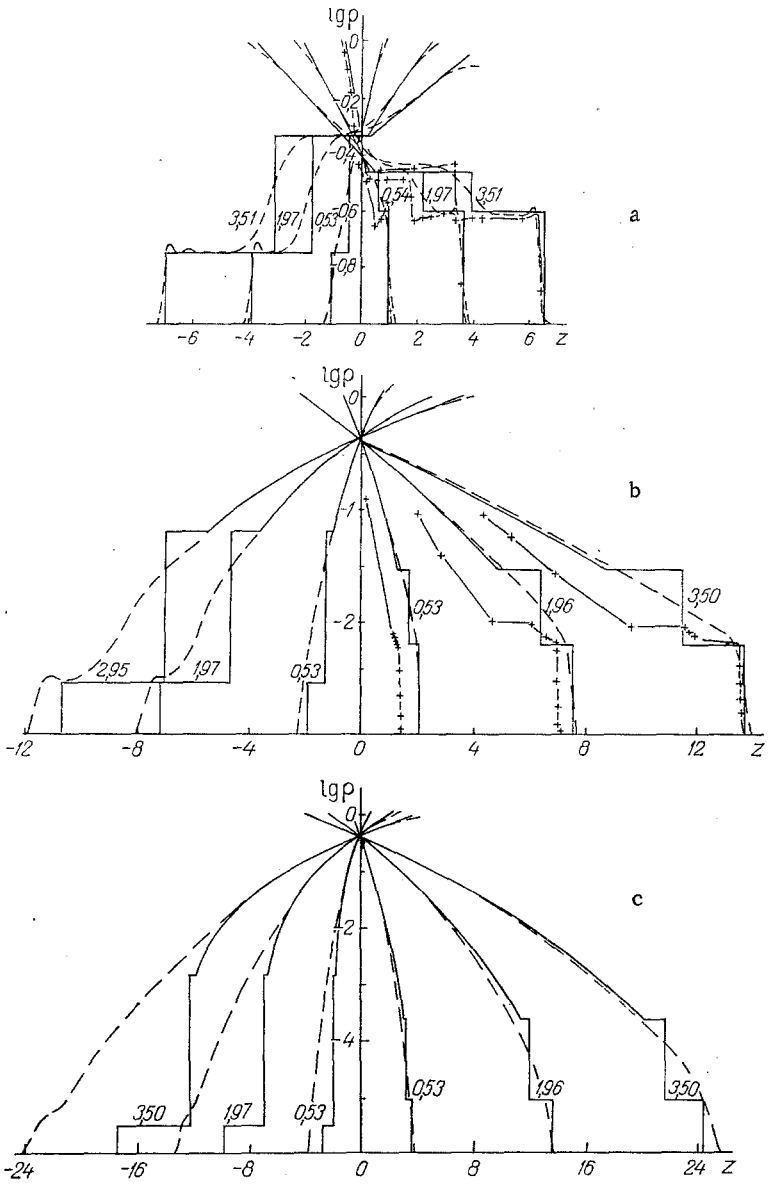

Fig. 3. Accurate (solid lines) and calculated (dashed lines) density profiles at successive instants for gas with $\gamma = 5/3$ (to the left of the axes of ordinates) and for gas with $\gamma = 1.2$ (right): a) $p/p_0 = 10$; b) 10^3; c) 10^6. Crosses denote the curves calculated by Lagrange's procedure [17].

It can be seen from Fig. 3a-c that the difference approach yields a blurred zone of contact discontinuity because of its numerical diffusion. Since gas with $\gamma = 5/3$ has a larger density gradient at the contact (it is being less compressed in the shock wave) than a monatomic gas, the numerical diffusion in the former case is greater than in the latter. Since in the case under consideration, the contact boundary represents a piston [16] generating the shock wave, the errors in calculaing the position of this piston obviously affect the accuracy of the calculation both of the shock wave and of the suction wave.

We want to point out that regardless of the strong numerical diffusion, Euler's method of large particles yields more accurate results of calculating the problem under examination than the Lagrange method of first order of accuracy [17] in which there is no numerical diffusion at all. It can be seen from Fig. 3a, b that the accuracy of the calculation obtained by the method of [17] is low because of the bad approximation of the contact region; this is caused by the overexpansion of the gas in the rarefaction wave. To eliminate this shortcoming, special measures have to be taken, e.g., the number of cells has to be permanently increased

in the calculation process in regions that are subject to strong tension in the direction of the Euler coordinate.

Let us briefly examine the special features of the calculation of two-dimensional disappearance of discontinuity from a pipe to the surrounding space. Two-dimensional flow coincides with unidimensional flow only at a fairly early stage of the process ($t \sim 0.17$), when the shock wave has passed about two pipe diameters. Then the flow begins to reform, gradually changing to the structure of an incompletely widened sonic jet, the shock wave changes its amplitude, behind it a section of low pressure forms in which the mass velocity and the Mach number are maximal. On account of the larger γ, the widening of the flow in space and the mentioned rearrangement of the structure occur earlier in a monatomic gas. Thus the flows in the unidimensional and the two-dimensional cases of the problem under examination differ substantially, and in the latter case the accuracy of the calculation as a whole depends largely on the accuracy of the calculation of the initial stage of the process. This is confirmed by the investigation of the convergence of the numerical results to the accurate solution by calculations on different grids with the number of cells in the pipe 20×4, 20×16, 20×32 along the z and r axes, respectively. By the instant $t = 3.5$ the shock wave has passed about 20 pipe diameters, but the discrepancy between the results on the second and third grids and the results obtained with the first grid still remains.

Thus, the study of non-steady-state gas flows provides an idea of the influence of the initial parameters of the problem on the accuracy of the obtained results and enables us to choose a substantiated approach to the evaluation of the accuracy of calculating the processes of formation of gas jets and their outflow into a medium with low counterpressure. It also follows that in some cases the calculations of flows of a high-temperature gas (which is characterized by small values of γ) may lay claim to greater accuracy than the calculations of flows of cold gas with large adiabatic index. In solving spatial gasdynamic problems, it is also necessary to attain maximum accuracy of the calculation of the initial phase of the flow since errors in its description may increase with time or exert a fairly lengthy influence on the nature of the flow.

NOTATION

γ, adiabatic index of an ideal gas; t, time; r, radial coordinate; z, axial coordinate; u, mass flow rate of the gas; p, pressure; ρ, density; D, velocity of the shock front. Subscripts: 0, initial values for the cold gas; 3, initial values for the hot gas.

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EVOLUTION OF THE INHOMOGENEOUS STATE OF A FLUIDIZED BED

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The article examines the change of the characteristics of the dilute and dense phases over the height of a fluidized bed, caused by the regular gas exchange between them and by the process of bubble coalescence.

The intensity of exchange processes in apparatus with a fluidized bed is largely determined by the local hydrodynamic state in the given section of the bed; the local values of the hydrodynamic parameters may substantially differ from the respective mean values for the bed as a whole. It is therefore indispensable to have hydrodynamic models of fluidization which would make it possible to describe the dependence of such parameters at different points of the bed on the physical characteristics of the particles and of the gas as well as on the type of apparatus and the regime parameters of the process.

The existing approximate models are of an empirical nature and do not meet this requirement. Within the framework of the two-phase theory of fluidization [1], the bed is regarded as a spatially homogeneous system. The "bubble" models of fluidization, which came into use in connection with the modeling of the longitudinal agitation in the bed [2, 3], are also based on the ideas of the two-phase theory, and differences in the state of the bed at different levels above the distributor grid are viewed as the consequences of bubble coalescence of the dilute phase; the properties of the dense phase are then taken to be homogeneous.

The principal relationship $u=u_*+\varphi u_b$ of the two-phase theory was already criticized by Turner [4], and deviations from it were experimentally confirmed by Pyle and Harrison [5] and in subsequent works. Attempts to modify this theory by taking into account the relative gas stream though the bubbles while maintaining the principal assumption $u_d = u_*$ (so that $u = u_*(1+m\varphi)+\varphi u_b$, where m is a parameter), begun by Grace and Clift [6], were unsuccessful, as was demonstrated by Davidson and Harrison [7]. Investigations in which the inadequacy of the two-phase theory was explained by pointing out that in reality $u_d > u_*$ [8, 9] permitted a number of empirical conclusions but they did not lead to the establishment of dependences of a sufficiently general nature.

The main circumstance determining the difference between the filtration rate in the dense phase and the speed of minimum fluidization is the difference between the "initial" in-

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